Math Principles

Algebra

▸ How do you **MULTIPLY AND DIVIDE ROOTS?**

▸ To multiply powers with the same base, add the exponents: $x^5 \bullet x^6 = x^{5+6} = x^{11}$.

▸ To divide powers with the same base, subtract the exponents: $y^{15} / y^{10} = y^{15-10} = y^5$.

Algebra

‣ How do you **RAISE A POWER TO A POWER?**

‣ To raise a power to an exponent, multiply the exponents: $(x^3)^4$ $x^{3\,*4} = x^{12}$.

Algebra

- How do you **SIMPLIFY SQUARE ROOTS?**

- To simplify a square root, factor out the perfect squares under the radical, unsquare them and put the result in front.

- For example $\sqrt{45} = \sqrt{9} * \sqrt{5} = 3\sqrt{5}$

Algebra

▸ How do you ADD AND SUBTRACT ROOTS**?**

▸ You can add or subtract radical expressions only if the part under the radicals is the same.

▸ For example $3\sqrt{5} + 6\sqrt{5} = 9\sqrt{5}$

Algebra

- How do you **MULTIPLY OR DIVIDE ROOTS?**

- The product of square roots is equal to the square root of the product:

- The quotient of square roots is equal to the square root of the quotient:

Algebra

- How do you **EVALUATE AN EXPRESSION?**

- To evaluate an algebraic expression, plug in the values you are given for the unknowns and calculate according to PEMDAS.

- For example to find the value of $x^2 - 4x + 3$ when $x = 5$, plug in 5 for x :
- $(5)^2 - 4(5) + 3 = 25 - 20 + 3 = 8$

Algebra

▸ How do you **ADD AND SUBTRACT MONOMIALS?**

▸ AS long as terms have the same variable then they can be combined according to standard rules of combining like terms.

▸ For example $2x + 5x = 7x$
▸ But $2x + 5y \neq 7xy$, $2x + 5y = 2x + 5y$

Algebra

- How do you **ADD AND SUBTRACT POLYNOMIALS?**

- Like anything you again just combine like terms.

- For example $(x^2 + 5x + 12) - (7x + 5)$
- HINT – this is a typical trap $- (7x + 5)$ Remember to distribute the negative sign
- So $(x^2 + 5x + 12) - (7x + 5) = x^2 + (5x - 7x) (+12 - 5) = x^2 - 2x + 7$

Algebra

- How do you **MULTIPLY MONOMIALS?**

- To multiply monomials, multiply the coefficients and the variables separately.

- For example $(5x)(6x) = (5)(6)\ (x)(x) = 30x^2$

Algebra

- How do you **MULTIPLY BINOMIALS?**

- Use FOIL. First, Outer, Inner, Last then combine like terms

- For example
  $(x + 7) (x + 2) = x^2 + 2x + 7x + 14$
  $= x^2 + 9x + 14$

- HINT another common trap is $(x + 4)^2$ Remember to FOIL not just square each part so the trap answer is $x^2 + 16$ and the correct answer is $x^2 + 8x + 16$

Algebra

▸ How do you **MULTIPLY OTHER POLYNOMIALS?**

▸ If you are not multiplying binomials make sure you multiply each term in each expression by each one in the other and then combine like terms.

▸ For example $(x^2 + 5x + 5)(2x + 3) =$
$x^2(2x + 3) + 5x(2x + 3) + 5(2x + 3) =$
$2x^3 + 3x^2 + 10x^2 + 15x + 10x + 15 =$
$2x^3 + 13x^2 + 25x + 15$

Algebra

▸ Factoring Part I How do you **TAKE OUT A COMMON FACTOR?**

▸ A factor common to all terms of a polynomial can be factored out.

▸ For example if you are given $5x^2y + 10x$
 They both share 5x so
   $5x^2y + 10x = 5x(xy + 2)$

Algebra

▸ Factoring Part II How do you **FACTOR THE DIFFERENCE OF SQUARES?**

▸ Be sure you look for the difference of squares in any form the present it and remember the general rule.

$a^2 - b^2 = (a - b)(a + b)$

▸ For example $x^2 - 25 = (x - 5)(x + 5)$.

Algebra

▸ Factoring Part III **HOW DO YOU FACTOR THE SQUARE OF A BINOMIAL?**

▸ Remember and use the following formulas
$$a^2 + 2ab + b^2 = (a + b)^2$$

▸ $a^2 - 2ab + b^2 = (a - b)^2$

▸ $a^2 - b^2 = (a - b)(a + b)$

▸ For example, $x^2 - 8x + 16 = (x - 4)^2$.

Algebra

- Factoring IV How do you **FACTOR OTHER POLYNOMIALS (REVERSE FOIL)?**

- To factor a quadratic expression, think about what binomials you could use FOIL on to get that quadratic expression. In essence you unFOIL.

- For example to factor
  $x^2 - 1x - 12$ you start with $(x \quad) \ (x \quad)$
  then look at the sign on the last term, if it is negative then the two binomials will have different signs, positive they will be the same
  $(x - \quad) \ (x + \quad)$
  then look for numbers that will multiply to -12 and add to -1
  $x^2 - 1x - 12 \ = \ (x - 4) \ (x + 3)$

Algebra

▸ How do you **SIMPLIFY AN ALGEBRAIC FRACTION?**

▸ Algebraic fractions are just like normal fractions. Common factors can be crossed out to simplify. Remember that only things being multiplied can be crossed out.

▸ For example $\dfrac{x^2 + 6x + 8}{x^2 - 4} = \dfrac{(x + 4)\,(x + 2)}{(x + 2)\,(x - 2)}$

so $(x + 4) / (x + 2)$

Algebra

- How do you **SOLVE A LINEAR EQUATION?**

- Combine like terms as you isolate the variable.
  Remember that you can do anything you want to an equation if you do it to both sides.

- For example $-2x + 7 = 22 + 3x$ subtract 7 from both sides and then subtract 3x from both so $-5x = 15$ and $x = -3$

Algebra

▸ How do you solve **IN TERMS OF**?

▸ Solving "in terms of" just means isolating that variable on one side of the equation and putting everything else on the other – combining like terms as usual.

▸ For example to solve $5x + 7y + z = -3y + 11z$ in terms of x

▸ $5x = -10y + 10 z$

▸ $X = -2y + 2z$ or $2(z - y)$

Algebra

- How do you **TRANSLATIE ENGLISH INTO ALGEBRA?**

- Look for key words and turn one phrase at a time into algebra. Use parentheses where appropriate to preserve the order of operations. Remember that is means equals, of means multiply, product is multiplication as well, sum means addition.

- For example If a car salesman's pay is $1000 plus 20% of his sales and he makes $5000 how much did he sell?
- Pay is means 5000 = and 20% of means multiply
- $5000 = 1000 + .2S$
- $4000 = .2S$
- $20000 = S$

Algebra

- How do you **SOLVE A QUADRATIC EQUATION**?

- To solve a quadratic equation, put it in the $ax^2 + bx + c = 0$ form, factor the left side (if you can), and set each factor equal to 0 separately to get the two solutions.

- If you do not see factors then you can use the quadric formula to solve. This is rarely to never found on SAT and rarely on ACT. Do this by plugging the coefficients a, b, and c from
- $ax^2 + bx + c = 0$ into the formula:
- $-b +/- b2-4ac / 2a$

Algebra

- How do you **SOLVE A SYSTEM OF EQUATIONS?**

- If you have two separate equations you can solve for two variables. Combine the equations by addition or subtraction in a way that cancels out one variable.

- For example given the following two equations solve for x and y
- $4x + 7y = 10$ and $-2x + y = 4$
- $4x + 7y = 10$
- $2(-2x + y = 4) \rightarrow -4x + 2y = 8$ now add
- $9y = 18$ so $y = 2$ and from the original $x = -1$

Algebra

- How do you **SOLVE A SYSTEM OF EQUATIONS?**

- Make sure you understand what the question is asking. There are 3 main ways that systems are addressed on the SAT.

- Question asks for the solution(s) → set the equations equal and solve for the point of intersection
- Question says there is no solution to the system → this means the lines are parallel or for non linear system there is no intersection, solve so that the slopes are equal
- Question says there are infinite solutions → the two functions are the same, solve to make them completely equal

Algebra

- How do you **SOLVE AN ABSOLUTE VALUE PROBLEM**?

- First simplify the equation then solve for the equation and then the inverse of it as well. think about the two different cases.

- For example if $|x - 10| = 7$, create two equations
- $x - 10 = 7$ or $x - 10 = -7$
- So $x = 17$ or 3?

- HINT: When in doubt, try and plug in the answer choices

Algebra

- How do you **SOLVE INEQUALITIES?**

- Isolate the variable as you would if there was an equal sign in the equation. The only difference is when you multiply or divide by a negative you need to reverse the sign.

- For example $-3x -7 < -1$
- $-3x < 6$ so x > -2

Algebra

- How do you **GRAPH INEQUALITIES**?

- Graphs that show the range of values, use a thick black line on the number line, and at the end(s) of the range, use a solid circle if the point is included or an open circle if the point is not included.

Algebra

- How do you **SOLVE FUNCTIONS?**

- Start with terminology if $f(x) = x^2 + 5$
- Whatever is in parentheses is the x value
- The rest $f(x)$ and/or $x^2 + 5 = y$

- Anything given to you in parentheses plug it in for the x value.

Algebra

- How do you **SOLVE FUNCTIONS? PART II**

- There are two ways you will be asked to deal with functions

- 1. If given equations plug in following the rules from the previous slide
- 2. If given graphs pull points off of them following the rules from the previous slide.

Algebra

▸ ## How do you **WORK WITH SERIES AND SEQUENCES?**

	Arithmetic Progression	Geometric Progression
Sequence	$a, a+d, a+2d, \ldots\ldots, a+(n-1)d, \ldots$	$a, ar, ar^2, \ldots, ar^{(n-1)}, \ldots$
Common Difference or Ratio	Successive term – Preceding term Common difference $= d = a_2 - a_1$	Successive term/Preceding term Common ratio $= r = ar^{(n-1)}/ar^{(n-2)}$
General Term (nth Term)	$a_n = a + (n-1)d$	$a_n = ar^{(n-1)}$
nth term from the last term	$a_n = l - (n-1)d$	$a_n = 1/r^{(n-1)}$
Sum of first n terms	$s_n = n/2(2a + (n-1)d)$	$s_n = a(1 - r^n)/(1 - r)$ if $r < 1$ $s_n = a(r^n - 1)/(r - 1)$ if $r > 1$

Algebra

- How do you **SOLVE QUESTIONS WITH LOGS**

- Remember the basic properties of logs and that on a simple level

- When b is raised to the power of y is equal x:
- $b^y = x$
- Then the base b logarithm of x is equal to y:
- $\log_b(x) = y$

- For example when:
- $2^4 = 16$
- Then
- $\log_2(16) = 4$

Algebra

▸ How do you **SOLVE QUESTIONS WITH LOGS**

Rules of Logs
$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
$\log_b(x / y) = \log_b(x) - \log_b(y)$
$\log_b(x^y) = y \cdot \log_b(x)$
$\log_b(c) = 1 / \log_c(b)$
$\log_b(x) = \log_c(x) / \log_c(b)$